

MST124

TMA04

2018J

Covers units 9, 10 and 12

Cut-off date 1 May 2019

You can submit this TMA either by post to your tutor or electronically as a PDF file by using the University's online TMA/EMA service.

Before starting work on it, please read the document *Student guidance for preparing and submitting TMAs*, available from the Assessment area of the MST124 website.

The work that you submit should include your working as well as your final answers.

Your solutions should not involve the use of Maxima, except in those parts of questions where this is explicitly required or suggested. Your solutions should not involve the use of any other mathematical software.

Your work should be written in a good mathematical style, as described in Section 6 of Unit 1, and as demonstrated by the example and activity solutions in the study units. Five marks (referred to as good mathematical communication, or GMC, marks) on this TMA are allocated for how well you do this.

Your score out of 5 for GMC will be recorded against Question 13. You do not have to submit any work for Question 13.

Question 1 – 15 marks

You should be able to answer this question after studying Unit 9.

Give sufficient details of your working to make it clear that you have not needed to use any software or calculator that can perform matrix manipulation. For example, in part (a), show explicitly how you calculated at least one element of each matrix answer.

(a) Let

$$\mathbf{A} = \begin{pmatrix} 3 & -1 \\ 2 & 5 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 2 & -7 \\ -3 & 5 \\ 4 & -6 \end{pmatrix},$$

Evaluate each of the following expressions, if possible. Where evaluation is not possible, explain why not.

(i) $\mathbf{AB}$

(ii) $\mathbf{BA}$

(iii) $\mathbf{B} - 2\mathbf{A}$

(iv) $\mathbf{BA} - 2\mathbf{B}$

(v) $\mathbf{A}^2 + \mathbf{A}$

[8]

(b) Determine whether the following matrices are invertible, and in each case find the inverse if it exists.

(i) $\begin{pmatrix} 5 & -2 \\ 10 & -4 \end{pmatrix}$

(ii) $\begin{pmatrix} -3 & 7 \\ -6 & 5 \end{pmatrix}$

[3]

(c) Use an answer from part (b) to solve the following system of linear equations:

$$-3x + 7y = 15,$$

$$-6x + 5y = 3.$$

[4]

Question 2 – 10 marks

You should be able to answer this question after studying Unit 9.

You can use Maxima to do the matrix arithmetic in this question, or you can do it by hand. If you use Maxima, then you do not need to include a printout, but you must include sufficient details in your solution to make it clear what calculations Maxima has done.

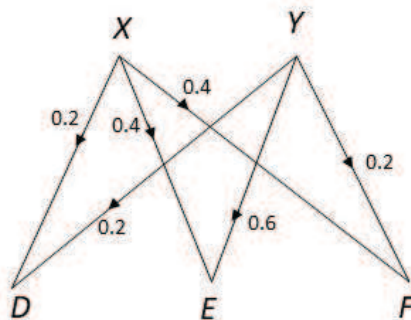
Two fitness centres, centre X and centre Y , provide courses for track athletes A , footballers B and cyclists C . During the year 2017:

60% of track athletes A went to centre X and 40% went to centre Y .

70% of footballers B went to centre X and 30% went to centre Y .

25% of cyclists C went to centre X and 75% went to centre Y .

- (a) (i) Draw a network diagram with input nodes A , B and C and output nodes X and Y , that represents which centre the track athletes, the footballers and the cyclists attended. [2]
- (ii) Write down the matrix that represents this network. [2]
- (iii) Use the matrix that you found in part (a)(ii) to determine how many people attended fitness centre X during 2017, if the numbers of track athletes, footballers and cyclists were 1200, 1400 and 1000 respectively. [2]
- (b) Each centre offers three courses. People can attend a course in strength and conditioning (D), a course in endurance (E) or a course in nutrition (F). The network diagram below represents the proportion of people who attended each course.



The matrix that represents this network is

$$\begin{pmatrix} 0.2 & 0.2 \\ 0.4 & 0.6 \\ 0.4 & 0.2 \end{pmatrix}.$$

- (i) Find the single matrix that represents the network obtained by combining the network in part (a)(i) with the network above. (That is, the combined network represents the number of people from each sports group, A , B , and C that attended the strength and conditioning course (D) or the endurance course (E) or the nutrition course (F).) [2]
- (ii) Find the numbers of people who attended the strength and conditioning course, the endurance course and the nutrition course, if the numbers of track athletes, footballers and cyclists who attended the centres were 1200, 1400 and 1000 respectively. [2]

Question 3 – 5 marks

You should be able to answer this question after studying Unit 9.

You should use Maxima to answer this question. Include a printout or screenshot of your Maxima worksheet with your solution. You are not expected to annotate your Maxima worksheet with explanation. However, remember for good mathematical communication you should present your answer clearly.

A farmer placed three orders with a grain supplier. The first order was for 4 bags of oats, 1 bag of barley and 3 bags of wheat, and the cost totalled £140. The second order was for 1 bag of oats, 2 bags of barley and 1 bag of wheat, and the cost totalled £90. The third order was for 3 bags of oats, 5 bags of barley and 2 bags of wheat, and the cost totalled £230. By modelling the problem as a system of three simultaneous linear equations, find the cost per bag of each type of grain.

[5]

Question 4 – 10 marks

You should be able to answer this question after studying Unit 10.

(a) For the infinite geometric sequence (x_n) whose first four terms are

$$3.6, \quad -9, \quad 22.5, \quad -56.25,$$

find the values of the first term a and the common ratio r , and write down a recurrence system for this sequence.

[4]

(b) Write down a closed form for this sequence.

[2]

(c) Calculate the 12th term of the sequence to three decimal places.

[1]

(d) Describe the long-term behaviour of the sequence. Justify your answer.

[3]

Question 5 – 5 marks

You should be able to answer this question after studying Unit 10.

On 1 January 2017, an art gallery had 50 pictures in stock. During January 2017, 3 more pictures were brought in. During each subsequent month, 2 more pictures were brought in than were brought in during the previous month. Assuming no pictures were sold, destroyed or given away, work out the total number of pictures in the art gallery on 31 March 2018.

[5]

Question 6 – 5 marks

You should be able to answer this question after studying Unit 10.

Find a fraction equivalent to the recurring decimal

$$0.564356435643\dots$$

[5]

Question 7 – 5 marks

You should be able to answer this question after studying Unit 10.

Find the sum of the infinite series

$$(a) \quad 5 - 5\left(\frac{4}{11}\right) + 5\left(\frac{4}{11}\right)^2 - 5\left(\frac{4}{11}\right)^3 \dots \quad [2]$$

$$(b) \quad \sum_{k=1}^{\infty} \left(\left(\frac{3}{5}\right)^k - \left(\frac{2}{9}\right)^k \right). \quad [3]$$

Question 8 – 5 marks

You should be able to answer this question after studying Unit 10.

Find the coefficient of x^4y^8 in the binomial expansion of

$$\left(\frac{1}{2}x - 2y\right)^{12}. \quad [5]$$

Question 9 – 8 marks

You should be able to answer this question after studying Unit 12.

(a) Given the complex numbers

$$z = 9 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$$

and

$$w = 3 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right),$$

$$\text{find } \frac{z}{w}. \quad [3]$$

(b) Use de Moivre's formula to find the Cartesian form of the complex number

$$(-1 + i)^9. \quad [5]$$

Question 10 – 10 marks

You should be able to answer this question after studying Unit 12.

You should use Maxima to answer both parts of this question. Include a printout or screenshot of your Maxima worksheets with your solution. You are not expected to annotate your Maxima worksheets with explanation. However, remember that for good mathematical communication you should present your answer clearly.

(a) Given the complex numbers

$$z = 5 \left(\cos \frac{3\pi}{11} + i \sin \frac{3\pi}{11} \right)$$

and

$$w = 2 \left(\cos \frac{4\pi}{11} + i \sin \frac{4\pi}{11} \right),$$

find the moduli and principal values of $z \times w$ and $\frac{z}{w}$. [5]

(b) Solve the equation

$$z^4 + 2z^3 + z + 2 = 0,$$

and plot the solutions in the complex plane. [5]

Question 11 – 7 marks

You should be able to answer this question after studying Unit 12.

Use the formula

$$\sin \theta = \frac{1}{2i} \left(e^{i\theta} - e^{-i\theta} \right)$$

to obtain the identity

$$\sin^5 \theta = \frac{1}{16} (\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta). \quad [7]$$

Question 12 – 10 marks

- (a) Think back over your experience of studying MST124: think about what has gone well, or what has not gone so well. Describe one change that you have made, or plan to make, to your studying, and describe how you hope that it will make your studying more effective. (If you think that no changes are necessary, then explain why.) [3]
- (b) Before completing this part of the question, we advise you to complete the Employability Skills Activity at <https://help.open.ac.uk/employability-skills-activity>.

This will help you to reflect on your skills and those developed in the workplace or through other activities. (This, of course, is a very useful activity to do before updating or starting to write a CV, or starting a LinkedIn profile to develop your professional network.)

While studying MST124, you will have developed a number of key skills, including some or all of the following:

- Thinking clearly.
- Paying attention to detail.
- Presenting a solution clearly.
- Being flexible, and approaching the same problem from different points of view.
- Seeking optimal solutions.
- Tackling a problem with confidence, even when the solution is not obvious.
- Writing mathematics clearly and effectively.
- Organising time and meeting deadlines.

Write a few sentences to summarise your key strengths and skills, and describe one or more areas that you would like to develop further as you progress through your studies. [3]

- (c) Before attempting this part of the question, we advise you to watch the video on Personal Development Planning at <https://help.open.ac.uk/pdp>.

Describe two things that you will do next to help you with your personal development. These could be closely connected with your own study of mathematics, e.g. learning how to type mathematics, or they could be more general, e.g. volunteering to help other people with mathematics.

It will be more helpful to you if you make a note to yourself of dates when you will achieve these by (you don't need to include these dates in your answer). [2]

- (d) Your answers should be clearly written and presented. You will be awarded up to 2 marks for the clarity and presentation of your answers. [2]

Question 13 – 5 marks

Your score out of 5 marks for good mathematical communication throughout TMA 04 will be recorded under Question 13.

Careers advice

If you are studying for career purposes and would like to find out more about career options, the Careers Advisory Service website (<https://help.open.ac.uk/topic/careers>) has a lot of information. In particular, you may like to look at the following page:

- Job seeking – job-finding strategies using traditional and more creative methods, at <https://help.open.ac.uk/job-seeking>.

Additionally, you can explore industry body websites such as the Institute of Mathematics and Its Applications careers information, at www.mathscareers.org.uk/graduates.

You may also wish to explore job sectors and roles via www.prospects.ac.uk.
